Lecture 12 – Chapt 9  Maxwell Relations, DNA Thermodynamics

Announce:
Exam times look like some time in 8-11:30am and also 1-3:30pm take care of almost everyone. (Rosa) HW5 is up now – due Friday – it IS on the exam. I’ll have answers up before I go on Friday.

Outline:
Maxwell relations
\[
\left( \frac{\partial U}{\partial V} \right)_T
\]
Single \(\Rightarrow\) double stranded DNA
Actinomycin - DNA complex

review:
\[U(S,V) \quad H(S,p) \quad F(T,V) \quad G(T,p)\]
and these lead us to 4 differential equations such as
\[
dG = \left( \frac{\partial G}{\partial p} \right)_T \, dp + \left( \frac{\partial G}{\partial T} \right)_p \, dT
\]
which relate back to our 4 fundamental equations such as
\[dG = Vdp - SdT\]
and therefore give us 8 relationships like
\[\left( \frac{\partial G}{\partial p} \right)_T = V \quad \left( \frac{\partial G}{\partial T} \right)_p = -S\]
Measuring one of the energies as a function of its natural variables allows us to calculate all other thermodynamics quantities.

Hydrophobic Effect
Processes like rubber band relaxation, ethane partitioning into cyclohexane vs. water and protein binding are driven by entropy, not enthalpy. One must consider both (i.e. free energy) when thinking about driving forces.

Remember that I asked you to think about why putting two proteins together into one could be good for entropy. To illustrate this, let’s look at ethane.

\[
\text{C}_2\text{H}_6 \text{ (cyclohexane)} \rightarrow \text{C}_2\text{H}_6 \text{ (aq)}
\]
\[\Delta H: \text{what do you expect?} \quad \text{for this process it is actually slightly favorable} \]
\[(\text{dispersion/VDW forces similar, slight dipole-induced dipole advanate to H}_2\text{O})\]
\[ \Delta H = -9.2 \text{ kJ/mol} \]

\[ \Delta S: \text{ what do you think? very negative.} \]

\[ \Delta S = -83.6 \text{ J/K-mol} \rightarrow -24.9 \text{kJ/mol at 298 K} \]

The overall \( \Delta G \), then, is driven positive by the entropy. **Why would dissolving ethane in water be bad for entropy?** This is because the water molecules form many H-bonds with each other. But, ethane can’t form any H-bonds, so if the water molecules are going to maintain all their H-bonds, the hydrophobic ethane forces them to into very restricted positions \( \Delta S < 0 \).

**What if we extend this thinking to two hydrophobic things dissolved in water?** They will both tend to get away from water to minimize the total hydrophobic surface area.

So, it isn’t that hydrophobic things attract each other, it’s that they all try very hard to get far away from water.

**Maxwell Relations**

Establishing relationships between the variables (like we did last time) is important because it gives us flexibility to measure what is convenient and relate it to everything else.

For instance, our eight basic quantities: \( p, V, T, U, H, A, G, \) and \( S \) lead to 336 possible slopes like: (there are \( 7 \times 6 \) possible ways to arrange variable in the denominator and the variable that is constant for each of the 8 quantities.)

\[
\left( \frac{\partial H}{\partial T} \right)_p = C_p \quad \left( \frac{\partial F}{\partial V} \right)_T = -p
\]

Note that each of the 8 quantities is a state function. **So each of these 336 slopes are?** also state functions.

If we open up our range of thermodynamic quantities to include things like changes in composition, surface area, electric, magnetic, or gravitational fields, the number of possible equations is about \( 10^{11} \).

Thus, we should probably figure out a general method of derivation rather than memorizing!! This will lead us to what are called the Maxwell relations.

Let’s start by looking at \( U \). **What is our fundamental equation for \( U \)?**

\[ dU = TdS - pdV \]

How did we start our derivation of the relationships from last time? wrote down total differential of \( U(S,V) \)

\[ dU = \left( \frac{\partial U}{\partial S} \right)_V dS + \left( \frac{\partial U}{\partial V} \right)_S dV \]
Since U is a state function, we must be able to write dU as an exact differential.

Recall that for an exact differential:

$$df(x, y) = \left( \frac{\partial f}{\partial x} \right)_y dx + \left( \frac{\partial f}{\partial y} \right)_x dy$$

is exact if

$$\frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right)_x = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right)_y$$

So, what does this mean for U?

$$dU$$ can only be exact if:

$$\frac{\partial U}{\partial V} = -\frac{\partial P}{\partial S}$$

This is a Maxwell relation. Note that they are named for James Clerk Maxwell of what fame? Maxwell Equations fame (from E&M), which you should know are incredibly important in the development of a quantum description of light as well as general and special relativity.

Not surprisingly, our other three fundamental equations lead to three more relations:

$$\left( \frac{\partial T}{\partial P} \right)_S = \left( \frac{\partial V}{\partial S} \right)_P$$

from

$$dH = TdS + VdP$$

$$\left( \frac{\partial P}{\partial T} \right)_V = \left( \frac{\partial S}{\partial V} \right)_T$$

from

$$dA = -pdV + SdT$$

$$\left( \frac{\partial V}{\partial T} \right)_P = -\left( \frac{\partial S}{\partial P} \right)_T$$

from

$$dG = VdP - SdT$$

Now we will explore one of these relations.

**Volume Dependence of U**

Think about an ideal gas. **How strong are the interactions between particles?** Zippo! So, if we have some container of ideal gas and shrink the volume, how should that affect the energy? It shouldn’t.

**How do we say that mathematically?** $$\frac{\partial U}{\partial V} = 0$$ for an ideal gas.

**What is our argument?** No interactions – so # of collisions doesn’t matter.

Let’s prove it.

Starting with $$dU = TdS - pdV$$ and dividing through by $$dV$$ at constant $$T$$ we get
\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T - p \left( \frac{\partial V}{\partial V} \right)_T \quad \text{using the appropriate Maxwell relation and}
\]
\[
\left( \frac{\partial V}{\partial V} \right)_T = 1
\]
\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V - p \quad \text{now plug in ideal gas equation } p = nRT/V
\]
\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{nR}{V} \right) - p
\]
\[
\left( \frac{\partial U}{\partial V} \right)_T = p - p = 0
\]

Now let’s derive \( \left( \frac{\partial U}{\partial V} \right)_T \) for a vdW gas (something you haven’t really seen much of yet)

\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V - p \quad \text{now plug in vdW expression: } p = \frac{RT}{V - b} - \frac{a}{V^2}
\]
\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{R}{V - b} \right) - \frac{RT}{V - b} + \frac{a}{V^2}
\]
\[
\left( \frac{\partial U}{\partial V} \right)_T = \frac{a}{V^2}
\]

\( U \) increases on expansion in direct proportion to the attractive term \( a \). What happens as the molar volumes gets large? implies larger mean intermolecular separations and the gas tends toward the ideal gas limit: as \( V \to \infty \), \( \left( \frac{\partial U}{\partial V} \right)_T \to 0 \).

Example: \( \text{N}_2 \) vs \( \text{H}_2\text{O} \) at 298 K and 1 atm \((V_m = 24.5 \text{ L mol}^{-1})\)

\( \text{N}_2 \ a = 1.408 \text{ L}^2 \text{ atm mol}^{-1} \) and \( \text{H}_2\text{O} \ a = 5.536 \text{ L}^2 \text{ atm mol}^{-1} \)
\[
\frac{\partial U}{\partial V} = 0.24 \text{ J L}^{-1} \text{ for N}_2 \text{ and } 0.92 \text{ J L}^{-1} \text{ for H}_2\text{O}
\]

We can see that this effect is very small for gases near atmospheric pressure. **Why is it larger in water (but still small)?** stronger intermolecular attraction (water is a hydrogen bonding molecule)

Speaking of water, let’s shift gears a bit to some important processes that do on in water.

First, note that Dill gets into a couple things (partial molar volumes and Gibbs-Duhem relationships) that we’ll do later. So, don’t worry you won’t miss out on them.

**Nucleic acid binding:**

Last time when we did the rubber bands and hydrophobicity, we talked about how solvent entropy can favor the attraction of hydrophobic things to each other. **Do you think this is the case for nucleic acid binding?** Probably not, because nucleic acids are very polar.

Let’s consider the following reaction:

\[
\text{single strand DNA} \rightarrow \text{double strand DNA}
\]

**What do we expect for }\Delta S\text{?** Solvent interacts favorably with polar single strands, so there should be little push toward binding. Consequently, }\Delta S\text{ negative for single }\rightarrow\text{ double strand}

**So, how does binding occur?** }\Delta H\text{ must be negative.}

**What drives negative }\Delta H\text{?** H-bonding seems likely

**Anyone heard of Chargaff’s rule?** It says that }\chi_A = \chi_T\text{ and }\chi_C = \chi_G\text{. This came from experimental evidence, but we know now that }\text{the molecular basis of this rule is?}\text{ Watson-Crick base pairing.}
Each hydrogen bond accounts for roughly 5 kJ/mol of favorable electrostatic interaction (relative to interaction with water)
But this is only about 1/3 of the total $\Delta H$. **What is the other 2/3?** Base stacking – dispersive interactions

So, $\Delta G = \Delta H - T\Delta S$

$\Delta H \approx -35$ kJ/mol/base pair
$\Delta S \approx -88$ J/K·mol/base pair

**What happens at high temp?** $|T\Delta S| > |\Delta H|$

$\rightarrow \Delta G$ is positive and DNA is all single strand

**What about low temp?** $|T\Delta S| < |\Delta H|$

$\rightarrow \Delta G$ is negative and DNA is all double strand

The details of this transition may appear in some future quiz or exam, but I can give away that $\Delta G$ is negative at normal body temperature (surprise surprise).

Let’s look at a slightly different DNA binding event in detail.

**Actinomycin - DNA complex**

**What is actinomycin?** chemotherapy drug – not surprisingly it has severe side effects. **Why?**
It is not at all selective. In fact it is a terrible poison, but it kills tumor cells slightly more effectively than healthy cells just because more bloodflow means that the concentration of drug is usually higher in tumors.

**Anyone know how it works?** It blocks transcription by plugging up and distorting the DNA. Because tumor cells reproduce faster they are more susceptible to damaging the transcription machinery.

It forms hydrogen bonds to guanine and it intercalates into its aromatic ring. (show nice chime webpage from UCSF)
In this case, binding is enthalpy-driven. Entropy is actually positive for binding (the aromatic ring system is fairly hydrophobic) but is a much smaller effect than enthalpy at room temperature.