Lecture 33 – Chapter 22, Sections 1-2
Nuclear Stability and Decay

• Energy Barriers
• Types of Decay
• Nuclear Decay Kinetics
Nuclear Chemistry
Nuclei Review

• Nucleons: protons and neutrons
• Atomic number – number of protons – identifies element
• Mass number – protons + neutrons

• Nuclei held together by the *strong nuclear force*
  – This force is incredibly strong – huge interaction energy
  – So much E is released that there is a measurable mass change

\[ E = mc^2 \]
\[ \Delta E = (\Delta m)c^2 \]
\[ \Delta E = \Delta m \times (8.988 \times 10^{10} \text{ kJ/g}) \]
Binding Energy Example

- Common isotope of helium has 2 neutrons and 2 protons.
- It has an isotopic molar mass of 4.00260 g/mol
  - Isotopic molar mass is exact mass of this isotope, including electrons

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Charge (10^{-19}) C</th>
<th>Mass (10^{-27}) kg</th>
<th>Molar Mass (g/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>p</td>
<td>+1.60218</td>
<td>1.672 622</td>
<td>1.007 276</td>
</tr>
<tr>
<td>Neutron</td>
<td>n</td>
<td>0</td>
<td>1.674 927</td>
<td>1.008 665</td>
</tr>
<tr>
<td>Electron</td>
<td>e</td>
<td>-1.60218</td>
<td>0.000 911</td>
<td>5.486 \times 10^{-4}</td>
</tr>
</tbody>
</table>

- Lone particle masses:

\[
2(1.007276) + 2(1.008665) + 2(0.0005486) = 4.0329792 \text{ g/mol}
\]

\[
\Delta E = \left(4.00260 - 4.0329792\right)\frac{g}{mol} \left(8.988 \times 10^{10} \frac{kJ}{g}\right)
\]

\[
\Delta E = -2.73048 \times 10^9 \text{ kJ}
\]
$^3$He has an isotopic mass of 3.0160293191 g/mol. What is the total binding energy of this nucleus?

$$\Delta E = \Delta m \times (8.988 \times 10^{10} \text{ kJ/g})$$

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25% 1. $-7.4464 \times 10^8$ kJ/mol
25% 2. $-8.2017 \times 10^8$ kJ/mol
25% 3. $-8.6948 \times 10^8$ kJ/mol
25% 4. $-1.8206 \times 10^{11}$ kJ/mol
Stability

• If it is so energetically favorable to add nucleons to a nuclide, why doesn’t every nucleus just keep getting bigger?

• Electrostatics! \[ E_{\text{electrostatic}} = k \frac{q_1 q_2}{r} \]

• For nuclei \( r \) is very, very small
• So, this presents a tremendous barrier to an incoming charged nuclide

• Neutrons are a little more complicated.
Who’s Law is This?

\[ E_{\text{electrostatic}} = k \frac{q_1 q_2}{r} \]

- 20% 1. Coulomb’s
- 20% 2. Einstein’s
- 20% 3. Joule’s
- 20% 4. Krueger’s
- 20% 5. Newton’s
More Stability

- Large nuclei have more stable isotopes
- H and He have two, F one
- Sn has ten
- # protons ~ # neutrons for small nuclei
- # neutrons a bit greater than # protons for larger nuclei

<table>
<thead>
<tr>
<th>Protons</th>
<th>Neutrons</th>
<th>Stable Nuclides</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>Even</td>
<td>154</td>
<td>58.8</td>
</tr>
<tr>
<td>Even</td>
<td>Odd</td>
<td>53</td>
<td>20.2</td>
</tr>
<tr>
<td>Odd</td>
<td>Even</td>
<td>50</td>
<td>19.1</td>
</tr>
<tr>
<td>Odd</td>
<td>Odd</td>
<td>5</td>
<td>1.9</td>
</tr>
</tbody>
</table>
The Middle Way

- The strong nuclear force is many-body. This means that 3 particles (per particle) are actually held together more strongly than two particles (per particle).
  - Bottom line is that larger nuclei are more stable (per particle) than smaller nuclei
- However, large nuclei become unstable because of Coulombic repulsion.
- So, there is a middle ground where these effects balance
  - Iron is the most stable nucleus.
Iron = Stable

- Smaller nuclei merge together to make larger ones
  - FUSION
- Larger nuclei fall apart to become smaller
  - FISSION
Nuclear Decay

• When nuclei spontaneously change from one nuclide to another, we call this **decay**
• Nuclides that are particularly unstable are **radioactive**
• Nuclear reactions are very similar to chemical reactions
  – Mass Number is conserved (protons + neutrons), though mass is not because of $E = mc^2$.
  – Charge is conserved

• For instance:

$$^0_1n \rightarrow ^1_1p + ^0_{-1}e$$

  – Note that sum of superscripts of reactants = products (Mass Number)
  – And sum of subscripts of reactants = products (Charge)
Nuclear Decay Processes

- Variety of processes are possible when nuclei decay
- Some involve particle emission
- Some involve particle absorption
- Some involve energy emission

<table>
<thead>
<tr>
<th>Type of Decay</th>
<th>Cause of Instability</th>
<th>Emission</th>
<th>$\Delta Z$ of Emitter</th>
<th>$\Delta A$ of Emitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ emission</td>
<td>Too massive ($Z &gt; 83$)</td>
<td>$\alpha = \frac{4}{3}$He</td>
<td>$-2$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$\beta$ emission</td>
<td>$N/Z$ too large</td>
<td>$\beta = -\frac{1}{3}$e</td>
<td>$+1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\beta^+$ emission</td>
<td>$N/Z$ too small</td>
<td>$\beta^+ = +\frac{1}{3}$e</td>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>Electron capture</td>
<td>$N/Z$ too small</td>
<td>X-ray photon</td>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\gamma$ emission</td>
<td>Excited nucleus</td>
<td>$\gamma$-ray photon</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
Alpha Radiation

- Alpha particles are energetic helium nuclei \((^4_2\text{He}^{2+})\)
- Very heavy positively charged particles
- Not too dangerous
- Can be stopped by fabric or a couple of sheets of paper

- Mass number of emitter decreases by 4
- Atomic number of emitter decreases by 2
- Occurs to nuclei that are unstable because they are too large (too many total particles)

\[
\begin{align*}
{^{222}_{86}}\text{Rn} & \rightarrow {^{218}_{84}}\text{Po} + {^4_2}\alpha \\
{^{218}_{84}}\text{Po} & \rightarrow {^{214}_{82}}\text{Pb} + {^4_2}\alpha
\end{align*}
\]
Belt of Stability

- Too many neutrons
- Belt of stability
- Too many protons
- \( ^{208}_{82} \text{Pb} \left( \frac{N}{Z} = 1.54 \right) \)
- \( ^{40}_{20} \text{Ca} \left( \frac{N}{Z} = 1.00 \right) \)

Legend:
- Black: Stable
- Orange: \( \alpha \) emitter
- Light blue: \( \beta \) emitter
- Light green: \( e^- \) capture and/or positron \( \beta^+ \) emitter

Graph:
- Number of neutrons, \( N \) vs. Number of protons, \( Z \)
- High \( N/Z \) ratio vs. Low \( N/Z \) ratio

The graph illustrates the stability of nuclides as a function of their neutron-to-proton ratio, highlighting the belt of stability where nuclides are most stable.
Beta Radiation

- Beta particles are electrons ($^0_{-1}\beta^-$)
- Very penetrating
- Beta decay occurs to nuclei that lie above the belt of stability.
  - These nuclei have ‘too many’ neutrons
  - Beta decay converts neutron to proton

\[
{}^1_0n \rightarrow {}^1_1p + {}^0_{-1}\beta^-
\]

\[
{}^3_1H \rightarrow {}^3_2He + {}^0_{-1}\beta^-
\]

\[
{}^{208}_{79}Au \rightarrow {}^{208}_{80}Hg + {}^0_{-1}\beta^-
\]
Belt of Stability

- Too many neutrons
- Too many protons
- $^{208}_{82} \text{Pb} \left( \frac{N}{Z} = 1.54 \right)$
- $^{40}_{20} \text{Ca} \left( \frac{N}{Z} = 1.00 \right)$

- Stable
- $\alpha$ emitter
- $\beta$ emitter
- $e^-$ capture and/or positron $\beta^+$ emitter

High $N/Z$ ratio
Low $N/Z$ ratio
Positron Radiation

- Positrons are anti-electrons \((^0_{+1}\beta^+)^\)
- Very penetrating
- Positron decay occurs to nuclei that lie below the belt of stability.
  - These nuclei have ‘too many’ protons
  - Positron decay converts proton to neutron

\[
^1_1 p \rightarrow ^1_0 n + ^0_1 \beta^+
\]

\[
^{18}_9 F \rightarrow ^{18}_8 O + ^0_1 \beta^+
\]

\[
^{144}_{63} Nd \rightarrow ^{144}_{62} Eu + ^0_1 \beta^+
\]
Belt of Stability

- Too many neutrons
- Too many protons
- Stable
- $\alpha$ emitter
- $\beta$ emitter
- $e^-$ capture and/or positron $\beta^+$ emitter
More Positron

- Positrons are not usually observed directly
- When a positron and electron run into each other they annihilate
  - Usually there are lots of electrons around, so this happens right away
  - Annihilation destroys both particles (no more mass), but releases loads of energy
  - Energy released as two gamma rays, each of $9.87 \times 10^7$ kJ/mol
Electron Capture

• Absorption of electron by the nucleus
• Electron removed from 1s orbital – leaves a ‘hole’
• This hole is filled by an outer electron falling down
  – Electron releases energy as an X-ray photon
• Like positron decay, electron capture occurs to nuclei that lie below the belt of stability.
  – Electron capture converts proton to neutron

\[ ^{26}_{13}Al + ^0_{-1}e \rightarrow ^{26}_{12}Mg \]

\[ ^{26}_{12}Mg^* \rightarrow ^{26}_{12}Mg + h\nu \quad _{X-ray} \]
Belt of Stability

The diagram illustrates the belt of stability on the nuclear chart. It shows the number of neutrons (N) versus the number of protons (Z) for different elements. The belt of stability is defined by the line N = Z, which indicates a balance between neutrons and protons. Elements outside this line are unstable and undergo radioactive decay to reach the belt of stability. The diagram includes examples of stable and unstable nuclei, such as $^{208}_{82}$Pb (N/Z = 1.54) and $^{40}_{20}$Ca (N/Z = 1.00). Elements with too many neutrons or too many protons are unstable and may emit particles like alpha ($\alpha$) or beta ($\beta$) particles, or undergo electron capture and/or positron decay ($\beta^+$).
Gamma Radiation

- Electromagnetic photons of **very** high energy
- Very penetrating → can pass through the human body
- Require thick layers of lead or concrete to be stopped
- Emitted from nuclei that have excess energy
  - Usually from products of other nuclear decay

\[
\begin{align*}
^{226}_{88} Ra & \rightarrow ^{222m}_{86} Rn + ^{4}_2 \alpha \\
^{222m}_{86} Rn & \rightarrow ^{222}_{86} Rn + \gamma \\
^{222}_{86} Rn & \rightarrow ^{218m}_{84} Po + ^{4}_2 \alpha \\
^{218m}_{84} Po & \rightarrow ^{218}_{84} Po + \gamma \\
\end{align*}
\]

Sometimes metastable (m) is denoted with a * meaning just ‘excited’
Identify the Product

$$^{214}_{84}Po \rightarrow ? + \alpha$$

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>1. Polonium-210</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>2. Lead-212</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>20%</strong></td>
<td>3. Lead-210</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>4. Mercury-212</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>5. Mercury-210</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Identify the product

\[ ^{210}_{82}Pb \rightarrow ^{210}_{83}Bi + \ ? \]

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>1. Alpha particle</td>
</tr>
<tr>
<td>20%</td>
<td>2. Beta particle</td>
</tr>
<tr>
<td>20%</td>
<td>3. Positron</td>
</tr>
<tr>
<td>20%</td>
<td>4. Gamma Ray</td>
</tr>
<tr>
<td>20%</td>
<td>5. X-Ray</td>
</tr>
</tbody>
</table>

1 2 3 4 5
Rates of Decay

- Recall from Chapt 15 that nuclear decay is unimolecular
  - First Order
  - Rate = $k \times$ concentration (or often # of nuclei)

\[
c = c_0 e^{-kt}
\]
\[
\ln\left(\frac{c_0}{c}\right) = kt
\]
\[
t_{\frac{1}{2}} = \frac{\ln 2}{k}
\]
\[
\ln\left(\frac{N_0}{N}\right) = \frac{t \ln 2}{t_{\frac{1}{2}}}
\]
Today

• Finish CAPA #19
• REVIEW REVIEW REVIEW!!!
• Go to Neckers lecture

Monday

• Please please review!
• Start CAPA #20 (due Wed)